Probabilistic behavior modeling of morphometric parameters for thermokarst plains with fluvial erosion in Cryolithozone

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Abstract—**This work aims to study the patterns of changing morphometric characteristics of thermokarst plains with fluvial erosion for different variants of their development. The research involved a series of essential morphometric parameters of the thermokarst plains with fluvial erosion, such as the number of lakes at a trial plot, a number of khasyreis at a trial plot, and areas of thermokarst lakes. The developed a probabilistic model is suitable for a homogenous area with a continuous generation of new thermokarst depressions ("asynchronous start") under a stable climate. The mathematical analysis of the model shows that after a long time of development with uneven occurrence and vanishing (drainage) of lakes, we get the stable share of the area covered by water and a particular distribution for the lakes' area – an integral exponential distribution. We chose 17 key sites in different regions of Western and Eastern Siberia and Canada for empirical testing of the model. The examination includes checking the conformity of the area samples to different types of distributions by the Pearson criterion. The test used aerial and satellite imagery of the two survey dates, including Corona archival photographs. The research revealed that at a majority of the key sites, the areas of the thermokarst lakes obey the integral-exponential distribution within homogeneous sections of the thermokarst plains with fluvial erosion in different natural environments. Moreover, the morphological pattern of the thermokarst plains with fluvial erosion is in a state of dynamic balance, and forecasting development and assessing natural risks should take it into account.**

I. Introduction

Many researchers deal with the cryolithozone landforms [1,2,3,4], but only a few of them analyze the behavior of morphometric parameters. The goal of this work is to study the patterns of changing morphometric characteristics of thermokarst plains with fluvial erosion.

Landscapes of thermokarst plains with fluvial erosion also include slight wavy subhorizontal areas covered by tundra vegetation, interspersed with lakes, and khasyreis (a khasyrei is a drained thermokarst lake), and crossed by a rare enough fluvial erosion network. The lakes of isometric often roundish shape are randomly scattered across the plain. Khasyreis are also isometric flat-bottomed and flattened peaty depressions covered with meadow or bog vegetation; like the lakes, they are randomly distributed across the plain (Fig.1).

The research involved a series of essential morphometric parameters of the thermokarst plains with fluvial erosion, such as a number of lakes at a trial plot, a number of khasyreis at a trial plot, and areas of thermokarst lakes. Thermokarst, thermoabrasion, and thermoerosion have a complex interrelation affect the area. As a result, new primary thermokarst depressions appear; different thermokarst depressions grow independently on each other as lakes (ponds) due to thermoabrasion; at a random moment, a lake can be drained by fluvial erosion and transforms into a khasyrei. At that, its growth stops because of the absence of water. These processes change the mentioned above morphometric parameters raising the following questions:

- What are the laws ruling the analyzed parameters?
- What are the dynamics of the analyzed parameters for a long time of development?
- Does the up-to-date climatic change influence the laws ruling the analyzed parameters?

II. Methods

We developed a probabilistic analytical model suitable for a homogenous area with a continuous generation of new thermokarst depressions ("asynchronous start") under a stable climate. The model belongs to the recent scientific branch called "the mathematical morphology of landscapes"[4].

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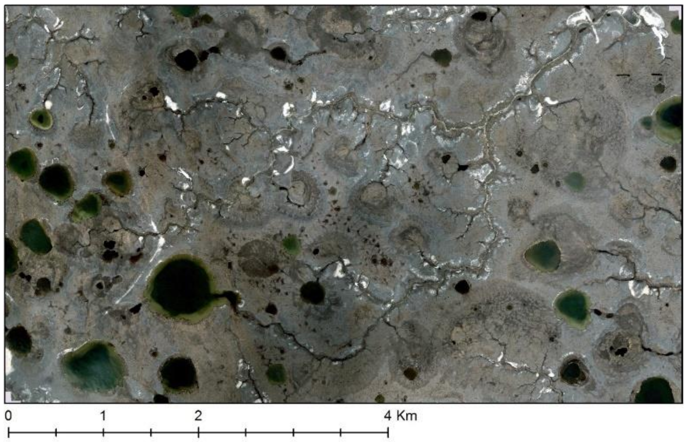

Figure 1. A typical image of a thermokarst plain with fluvial erosion on the remote sensing data.

The base of the model 1.0 includes the following underlying assumptions:

1. Thermokarst depressions (germs of thermokarst lakes) were appearing within a relatively short period independently across the different non-adjacent sites. At that, for small plots and time intervals, the probability of several depressions to occur is much less than that of a single depression.

2. The change of the radius of an appeared thermokarst depression is a random variable; it is independent of other lakes, and the growth rate is directly proportional to heat losses through the side surface of the lake basin.

3. In the course of its growth, a lake can transform into a khasyrei after draining by the erosion network; the probability of this does not depend on the development of other lakes; if it happens, the depression stops growing.

4. The appearances of new sources of fluvial erosion within a randomly selected area are random and independent events. At that, the probability of occurrence of more than one source is an infinitesimal of a higher-order than the probability of the appearance of one source.

5. The primary thermokarst depressions do not occur within already existing thermokarst lakes.

The following primary dependencies obtained earlier in the frames of the mathematical morphology of landscapes are valid [1,4]:

- The radius distribution for free growing thermokarst lakes since time t after lake emergence (lognormal distribution):

$$f_0(x,t) = \frac{1}{\sqrt{2\pi}\sigma x\sqrt{t}} e^{-\frac{(\ln x - at)^2}{2\sigma^2 t}} \tag{1}$$

where a, σ are distribution parameters, t is the time since the process has started.

- The distance distribution from the center of the growing lake to the nearest sources of erosion structure (stream) which stops the lake growth and transform it into a khasyrei, obeys the Rayleigh distribution

$$F(x) = 1 - e^{-\pi\gamma x^2} \tag{2}$$

where γ is an average density of the stream sources.

- The number of primary depressions on a free surface without lakes corresponds to the Poisson distribution.

The most uncomplicated characteristics include a number of lakes and a number of khasyreis within a random plot. Since the probability for a lake to transform into a khasyrei does not depend on its location, it is easy to show that the distributions of lakes and khasyreis within a test plot are Poisson at any moment.

The behavior of lakes' areas is more complicated. The density distribution of the lake radii at time t is equal to the ratio of the number of lakes of a given radius to the total number of lakes, taking into account the different times of their appearance and the probability not to transform to a khasyrei. The assumption of the model that primary depressions appear only outside the lakes means that a variable density of generation of initial depressions is equal to

$$\lambda_1(t) = \lambda[1 - P_l(t)] \tag{3}$$

where $P_l(t)$ is a share of water covered area, λ is a generation density of initial depressions within the free of lakes territory.

The function $P_l(t)$ is a fraction of the whole area occupied by lakes at time t and, as shown earlier [4], is related to the process parameters by the dependence

$$P_l(t) = 1 - e^{-\tau(t)\bar{s}(t)} \tag{4}$$

where $\bar{s}(t)$ is an average lake area at time *t*, $\tau(t)$ is an average number of lakes per unit area at time *t*.

The probability of a lake to remain the lake without transforming into a khasyrei depends on the lake radius distribution in the case of free growth if the distance to a fluvial source is more significant than the lake radius. The distance to the source fits the Rayleigh distribution (2). Thus, after simplification, the distribution density for the thermokarst lakes at time t is equal to

$$f(x,t) = \frac{e^{-\pi\gamma x^2}\int_0^t [1-P_l(u)] f_0(x,t-u)du}{\int_0^t [1-P_l(u)]\int_0^{+\infty} e^{-\pi\gamma x^2} f_0(x,t-u)dxdu} \quad (5)$$

Let us examine the area distribution of the thermokarst lakes after a long time of development, which we observe now. First, our model gives us an expression of the dynamics of a share of the area covered with water. If we take the average lake area from the above formula for the lake radius distribution, then involving equations (4) and (1) after simplifying and taking the logarithm we get the integral equation [5]

$$ln[1-P_l(u)] = -\pi\lambda\int_0^t [1-P_l(u)]\int_0^{+\infty} x^2 e^{-\pi\gamma x^2} f_0(x,t-u)dxdu \quad (6)$$

The function $P_l(t)$ is the solution of this equation. Using this integral equation, we can show that if the integral converges

$$I = \int_0^{+\infty}\int_0^{+\infty} x^2 e^{-\pi\gamma x^2} f_0(x,u)dxdu \quad (7)$$

and the solution of the equation (8) (at that, we can prove that this equation has a solution and the only one)does not exceed $(1-e^{-1})$ that is 0.63; then, there is a limit of function $P_l(t)$ if $t \to +\infty$. It is equal to the solution of the equation (P_l^*). The proof needs the construction of a pair of step functions that bound a function $P_l(t)$ from above and below [4,5].

$$\ln[1-P_l^*] = -\lambda\pi[1-P_l^*]I \quad (8)$$

Under these conditions, ensuring the existence of a limiting value of the lake area percentage, there is also a limiting distribution of the radii of lakes if $t \to +\infty$

$$f(x,\infty) = \frac{e^{-\pi\gamma x^2}\int_0^{+\infty} f_0(x,u)du}{\int_0^{+\infty}\int_0^{+\infty} e^{-\pi\gamma x^2} f_0(x,u)dxdu} \quad (9)$$

Using expression (1) for the density distribution of the lake radii in the case of free growth and calculating the upper integral as the Laplace transform, we obtain the area distribution (with ε area for the primary depressions)

$$f_{sl}(x,\infty) = -\frac{2}{xEi(-\gamma\varepsilon)} e^{-\gamma x}, x \ge \varepsilon \quad (10)$$

where

$$Ei(-x) = \int_{-\infty}^{x} \frac{e^{-u}}{u} du \quad (11)$$

is an integral exponential function, and thus, this distribution can be called the integral exponential distribution.

Thus, our model shows that after a long time of development, we obtain the stabilization of the total lake area and the integral exponential distribution of lake areas. The empirical testing of these analytical results involved 17 key sites within the thermokarst plains with fluvial erosion in different natural environments (Fig. 2).

Figure 2. A location scheme of the key sites of the thermokarst plains with fluvial erosion.

We used repeated remote sensing data. The first date of the survey comes from the archive Corona imagery (3-7 m/pix, 1965-1976) for eleven key sites. The recent imagery includes Sentinel 2A 2017-2018, Resurs-P, ICONOS, QuickBird, Worldview 2, Geoeye-1, 2008-2014 for the whole set of 17 key sites.

We found the Pearson criterion for the common distributions using the STATISTICA package, while we have to make a special software for the integral exponential distribution. Parameter ε is the minimal value of the sample, while γ resulted from the numerical solving the equations in the frame of this software package.

$$-\frac{1}{\gamma Ei(-\gamma\varepsilon)} e^{-\gamma\varepsilon} = \bar{s} \quad (12)$$

where $\bar{s}$ is an average lake area.

III. Discussion

The empirical data of the thermokarst lakes from the key sites include samples from 49 to 2108 lakes. The empirical distributions fit the integral exponential distribution (fig. 3 as an example) for ten key sites from 17 key sites (59%) of the second date and five key sites of 11 (45%) of the first date.

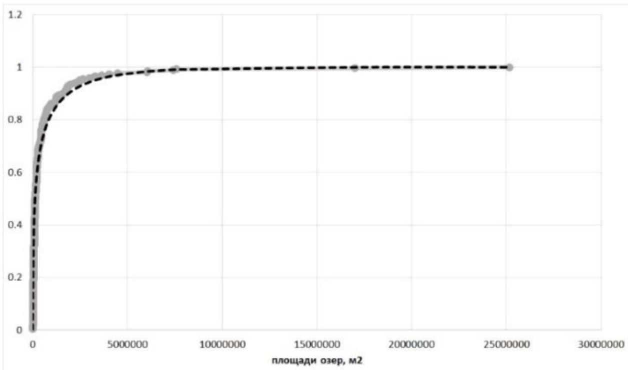

Figure 3. A graph demonstrating closeness between an empirical distribution and the integral exponential distribution (key site 28).

At the same time, the lognormal distribution characterizes area distribution of the thermokarst lakes at eight key sites of 17 of the second date. For three key sites of these 17, the empirical distributions of the thermokarst lakes' areas obey both the gamma and lognormal distributions. This situation corresponds to the synchronous start model [4] and can be explained by two reasons:

- At the first stage, the thermokarst plains with fluvial erosion are just lacustrine thermokarst plains since the probability of lake drainage was small due to their little size; this causes the lognormal distribution of the lakes' areas.
- The integral exponential distribution is a limit distribution at $t \to \infty$, while the time since the start of the thermokarst process is long but finite.

Interestingly, typical lacustrine thermokarst plains fit generally the lognormal distribution only [4].

We can explain the fact that the distribution of the lake areas fits neither the integral exponential nor the lognormal distribution for several key sites by the beginning change of these sections under the influence of climate change. Four of these key sites demonstrate a significant (by the Smirnov's criterion) difference in the distribution of lake areas for the two periods.

IV. Conclusions

The model fitting the asynchronous start and the lake growth rate proportional to heat losses through the side surface is relevant for most of the homogenous sections of thermokarst plains with fluvial erosion in different natural environments.

These sections have mostly integral exponential distribution of the thermokarst lake areas.

The morphological pattern of the thermokarst plains with fluvial erosion can be in the state of the dynamic balance; natural risk assessment and prognosis studies should take it into account.

There are the signs of a shift of the dynamic balance apparently due to climate change.

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