

The D8 implementation of the surface stream function

John Gallant[§]

CSIRO Land and Water
 Black Mountain Laboratories
 Clunies Ross St, Canberra ACT, Australia
[§] john.gallant@csiro.au

Abstract— The land surface stream function has the potential to simplify many tasks related to the analysis of surface flow paths, but a practical implementation is required to make it usable. Three implementations of varying complexity have been developed and the simplest, based on D8 flow paths, is described in detail.

While this simple method lacks fidelity to true flow paths it provides a usable approximation to the true stream function whenever areas anchored on stream channels are considered. To each cell, the method assigns a minimum and maximum stream function that can be used to delineate catchments and trace catchment boundaries without working through the interior of the catchment. It is not suitable for creating flow meshes because it ignores dispersal area.

I. STREAM FUNCTION IMPLEMENTATIONS

The concept of the land surface stream function [1] is very appealing and provides a novel and informative way of viewing catchment structure. But a practical implementation is required before it can be used with real data.

Producing a stream function map from a grid digital elevation model is not a simple task. The method must capture the locations of peaks, sinks and saddles, construct the various lines – divide, course, ridge and valley lines – and calculate stream function on a regular grid accounting for the discontinuities along divide and course lines.

The general approach is to calculate contributing area A and dispersal area D within each cell or on each cell edge and use the sum $A + D$ as the difference in stream function in the cell or across the cell edge. The stream function value can then be accumulated from one cell to the next starting at an arbitrary zero point and accounting for the discontinuities and merging of lines.

Three implementations have been developed with varying degrees of simplification. The simplest method adopts the

extreme simplification of forcing all slope lines to converge to a single line in each grid cell and uses the single-direction steepest descent to construct the slope lines between cells. Under this construction there are no dispersal areas and the stream function calculation closely follows the calculation of contributing area using the D8 method. Following the cell-to-cell connectivity in an orderly manner, each cell is assigned a minimum and maximum stream function value and the difference in values exactly equals the cell's D8 contributing area. Prior identification of peaks, sinks and saddles is not required in this method provided there are no depressions, which matches the typical application where contributing area is calculated. Peaks and ridge lines are considered to be located on cell corners and edges and their dispersal area is represented by the gap in stream function ranges for adjacent cells. While this extreme simplification prevents the proper construction of stream tubes and hillslope flow elements it still allows some of the most useful applications such as tracing catchment boundaries from points along streams and computing the hydrological significance of ridge lines.

The second method is a detailed implementation known as Terrae (outlined in [2]). It uses a novel method for computing A and D that assigns values to cell edges, allows convergence and divergence within cells and takes measures to control artificial dispersion. It identifies peaks, sinks and saddles and assigns dispersal and contributing areas to peaks and sinks respectively. Divide and course lines are explicitly constructed, and the magnitude of stream function discontinuity is assigned to each line using the peak and sink areas, accumulating values along chains of divide and course lines as needed. The stream function difference along each cell edge is $A + D$ plus the contributions from any discontinuities, with the sign defining positive or negative change in a clockwise direction. For each cell the sum of stream function differences around the four cell edges is zero except where the cell contains a peak or a sink. Starting at an arbitrary point a stream function value of zero is assigned to a cell corner and the difference in value along the edge, plus the change across any discontinuity, allows the stream function value to be

John Gallant (2020) The D8 implementation of the surface stream function:

in Massimiliano Alvioli, Ivan Marchesini, Laura Melelli & Peter Guth, eds., Proceedings of the Geomorphometry 2020 Conference, doi:10.30437/GEOMORPHOMETRY2020_61.

assigned to the corner at the far end of the edge. This process is repeated until all corners have been given a value. The method is complicated by the need to trace every divide and course line, including merging of lines along ridges and valleys, and to capture the discontinuity in stream function on every cell edge crossed by one of those lines.

The third method was inspired by the stark contrast in complexity for the first two methods. How can the very simple method capture the discontinuities at divide and course lines without having to explicitly identify them as Terrae does? The answer is that by following the tree structure of the steepest descent paths the simple method automatically steps around the discontinuities that are, as noted before, located on cell edges and corners. When slope line divergence and convergence are allowed, as in the full Terrae method, there is no simple tree structure to follow but the lines of discontinuity can still be forced to cell edges without much distortion of the stream function. This requires identification of peaks, sinks and saddles and those divide lines and course that contain discontinuities, as in Terrae, but there is no need to determine the change in stream function at the discontinuities. For this method to work, stream function values must be recorded at both ends of each cell edge so that the stream function value on an edge can take on different values on either side of the edge. After calculating *A* and *D* values for every cell edge the accumulation of stream function values can proceed as in Terrae but values are not propagated across divide and course lines that have discontinuities. Because a saddle creates discontinuities along either divide *or* course lines – never both – there is always a path to propagate stream function values around the end of discontinuities. When the full set of stream function values has been calculated the change in stream function across each discontinuity is automatically captured.

The simplest D8-based method is described here. The other methods will be described in papers currently in preparation and the implementations will be released with the papers.

II. THE D8 STREAM FUNCTION METHOD

In this method each cell is divided into 8 triangular segments and the stream function is calculated in units of 1/8 of cell area, so integer values can be used. Each cell is assigned a minimum and maximum stream function value.

Figure 1 shows a simple example for illustration of the method. The calculation starts at an arbitrary cell on a boundary (edge of DEM or coastline) that has an outflow, and the initial stream function value of 0 is assigned as the minimum value for that cell. In Figure 1 there is only one outflow cell, so that is the starting cell.

Within the cell being processed, each potential inflowing direction is considered in sequence in a counter-clockwise direction, starting from the outflow direction, and the stream function is incremented once for each change of direction. If an

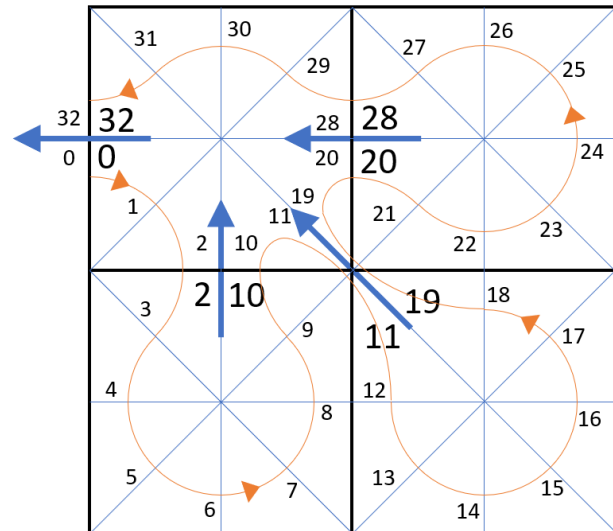


Figure 1. A simple D8 flow network showing flow paths (blue arrows) between cells (black squares). The cells are divided into eights representing the areas between each potential inflow direction (thin blue lines). The sequence of processing stream function is shown by the orange line and arrows. Stream function at each step is shown by the smaller numbers, while the larger numbers are the minimum and maximum stream function allocated to each cell.

inflow is found, the calculation shifts to the source cell and again starts by setting the cell’s minimum stream function value to the current value and setting the flow direction to the cell’s outflow direction.

When all eight inflowing directions have been considered and the flow direction matches the cells outflow direction, the cell’s maximum stream function is set to the current stream function value and processing shifts to the downstream cell with the flow direction set to the inflow direction from the cell that has just been finished.

When the processing finishes tracing the tree for the initial cell, which has an outflow that exits the DEM or land area, the boundary is traced in a clockwise direction looking for another outflow that has not already been processed. If there are multiple disconnected land areas (islands), the search for an unprocessed outflow should continue across the DEM and each separate boundary traced to find all outflows.

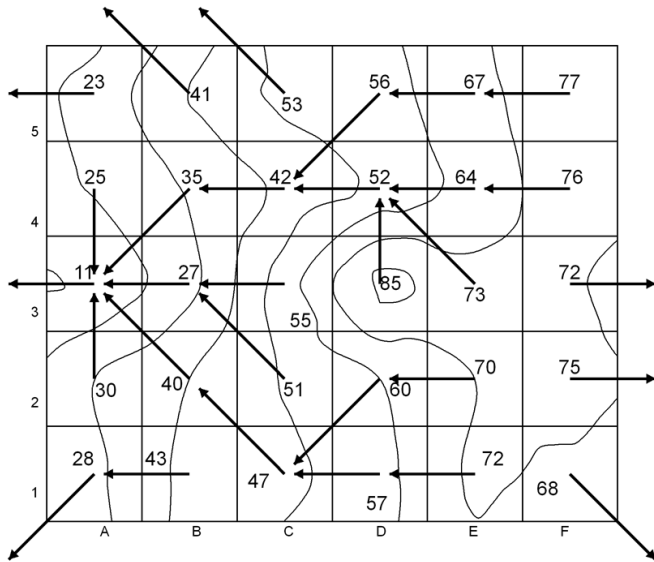


Figure 2. A synthetic example showing elevations, flow directions and contours (curved lines).

The path of processing and the intermediate stream function values are shown in Figure 1, as well as the minimum and maximum values assigned to each cell.

Figure 2 shows a larger area with a more complex flow network, labelled with elevations and showing contours. Most of the area is occupied by a single catchment that flows across the boundary of the DEM at cell A3. Figure 3 shows the stream function values for the same flow network.

III. STREAM FUNCTION APPLICATIONS

A. Contributing area

The difference in D8 stream function value between minimum and maximum is equal to the D8 contributing area of each cell, measured in units of 1/8th of the cell area. The stream function method takes only marginally more time to compute than contributing area and produces contributing area (in cells) as well as the two stream function results:

$$A = \frac{\psi_{max} - \psi_{min}}{8}$$

Note that this equation does not apply to stream function in general, only to this specific implementation where dispersal area is always zero due to the way flow paths are treated.

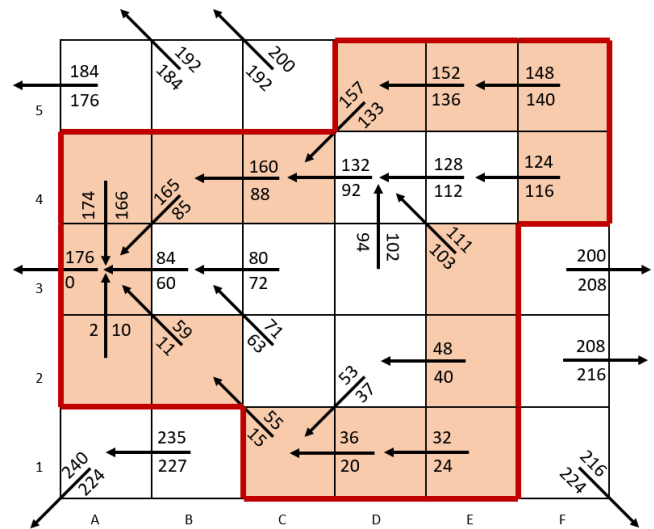


Figure 3. Minimum and maximum stream function values for the flow network of Figure 2. The shaded cells are the catchment boundary cells identified by the boundary tracing algorithm, and the catchment boundary itself is the outer edges of those cells, highlighted in red.

B. Catchment delineation

Given a cell A, any cell with stream function values within the range of that cell are inside the catchment of that cell. The catchment of cell A is comprised of all cells satisfying:

$$\psi_{min} > \psi_{min,A} \text{ and } \psi_{max} < \psi_{max,A}$$

This pair of conditions can be used to find all the cells that are within the catchment area of cell A, or to test whether an individual cell lies within cell A's catchment.

C. Measuring the hydrological significance of divide lines

The change in stream function across a divide line is a measure of the hydrological significance of the divide. For a minor divide where the slope lines either side of the divide converge after only a short distance, the difference in stream function is small. On a major divide, where the slope lines might not converge at all before they reach a coastline, the difference in stream function is large. This is similar to the maximum branch length described by Lindsay and Seibert [3] except that the difference in stream function value represents the area between the two slope lines rather than the length.

In the D8 implementation of stream function, the divide is between cells, so values in adjacent cells must be used, not the difference between maximum and minimum value in a single cell.

D. Tracing a catchment boundary

The D8 stream function supports tracing of catchment boundaries with a very simple algorithm. Starting at a catchment outlet, such as cell A3 in Figure 3, the minimum and maximum stream function values are noted. A current direction is set to the outflow direction for the starting cell. The adjacent cells are then examined, starting from the current direction and proceeding in a clockwise direction, looking for a cell that has stream function values within the range of the outlet cell. When such a cell is found, processing moves to that cell and it is marked as part of the catchment boundary. The current direction is reversed so that it points to the previous cell, and then rotated one step clockwise before continuing the examination of the adjacent cells. The process repeats until it arrives back at the starting cell. Figure 3 shows the cells identified by the boundary tracing algorithm starting at cell A3. The cells identified by this process are inside the catchment boundary and the boundary itself is the outer edges of those cells, identified by the heavy red line in Figure 3.

This algorithm is used by DamSite [4], a tool for identifying topographically favourable sites for constructing new dams. The DamSite method uses the set of catchment boundary cells identified using the D8 stream function to ensure that all saddle dams are accounted for, no matter how far from the main dam wall, when estimating the dam construction cost.

IV. CONCLUSIONS

The D8 implementation of the stream function is a very crude approximation to the full stream function because it completely ignores dispersal areas, but it captures many of the useful features of the stream function such as delineation of catchment areas and catchment boundaries. The algorithm to compute it is fast and takes little more computation than D8 contributing area.

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