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What does land surface curvature really mean?

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Abstract — **The complex development of geomorphometric theory has led to various concepts of land surface curvature (LSC), whose compatibility has not yet been systematically investigated. The definition, terminology and interpretation of LSCs show significant confusion and gaps reflected in various applications in geoscience modelling and prediction. Here we discuss the present situation, specify basic problems, and make initial suggestions for their solution.**

I. Introduction

Land surface curvatures (LSCs) are an important part of systems of geomorphometric variables [1]. For long, the characterisation of land surface by curvatures of contour line and slope line (i.e. plan and profile) was considered sufficient [2-5]. During the 1990s came an explosion of new definitions of LSCs. The introduction of particular new LSCs (tangential curvature [6], flow line curvature [7], and Laplacian as a 'curvature' [8]) was followed by two attempts to build a system of LSCs. The first system used directional derivatives of slope and aspect to derive all existing and two new curvatures – changes of slope in directions of contour and of slope line [9]. The second system draws from differential geometry of surfaces, and includes four curvatures long known in geometry (maximal, minimal, mean and total) as well as six new curvatures (unsphericity, difference, horizontal and vertical excess, total accumulation and total ring) [10]. This remains the most compact comprehensive system of LSCs and is well represented in three representative books devoted to digital terrain analysis [11-13].

Nevertheless different definitions of LSCs with the same name are frequently used in practice: plan and profile curvature [5], maximum and minimum curvature [14], mean curvature [15], and total curvature [16]. Moreover, further new curvatures or curvature-like variables have appear in the literature: curvedness [17], transverse and profile terrain curvature [18], slope of slope or slope of aspect (e.g. [19]), and a very strange variable simple termed 'curvature' [20]. GIS packages offer dozens of variously termed 'curvatures', frequently without clearly distinguishing between them. All this is a challenge for the geomorphometric community to discuss standardization of formal expression, terminology and labelling, elucidating systemic relationships and improving the interpretation of LSCs. Space here does not permit presentation of our proposals synthesizing contemporary knowledge and some new theoretical concepts into a new system of definition and interpretation of LSCs. Here we focus on the state of the art and outline some questions and principles important for the building of such a system. Discussion of the interpretation of curvatures in terms of attractors of land surface development is an example of a new research problem formulated on the basis of our approach.

II. Definitions

LSCs sensu stricto are the subject of this paper: they are local field-based geomorphometric variables defined by partial derivatives in the differentially small surrounding of a point [1]. There are other variables termed as 'curvature' that do not meet this requirement (e.g. [18, 20]). Two overlapping approaches to definition and derivation of LSCs exist. The first uses formulae from differential geometry to derive basic geometric curvatures defined by the inverted value of an osculating circle [4, 7, 10, 21]. Curvature of curves and normal curvatures of surface are most frequently used. However, the synthesizing concept of curvature of curves on surfaces, defining their normal and geodesic curvature as well as geodesic torsion (e.g. [22]) has not yet been systematically applied in geomorphometry. The second approach understands LSCs as 2nd directional derivatives of altitude or 1st directional derivatives of slope and aspect [3, 5, 19]. Some authors show the compatibility of both approaches [9, 23] but it is not absolute.

LSCs can be defined exactly by mathematical formulae in general or specific form. The most frequent general expression in Cartesian coordinates [4, 6, 7, 9, 10, 21] permits the simple comparison of particular variables. Specific computational expressions, using coefficients of interpolation polynomials [5, 8, 14, 24], is also frequent. Unifying the expression of all investigated LSCs in the Cartesian coordinate system produces more than 30 different equations, some of which are similar. Deeper understanding of their derivation and interpretation has permitted preparation of a new classification system containing all generally used LSCs, and expressing interrelationships.

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Shary [10] proved that three curvatures (mean, unsphericity and difference curvature) can be considered as independent components of LSC; and seven other geometric curvatures are combinations of the three. However, that is only one kind of relationship between LSCs. Some LSCs can be defined as slope (S) dependent transformations (subforms) of others (e.g. the well-known relationship between plan contour (p) and tangential curvature (t): p = t / sinS), and some curvatures are the same as others, but only in specific conditions (we label this 'imitation'). Combination of these various kinds of relationships with classification of the mathematical basis of particular curvatures is at the core of our suggested new system of LSCs.

A very frequent problem of LSC definition concerns signs of the defining equations. At first sight trivial, this problem (what is convex in mathematics, and what on the land surface) has far from trivial consequences. To be mathematically correct we should term hills as concave and dells as convex. Changing the direction of slope gradient permits correct definition (geographically as well as mathematically) of basic LSCs [25]: a problem remains, however, in definition of maximal and minimal curvatures. A simple change of signs is not effective here: maximal curvature in the geographical convention is described by minimal directional derivatives and vice versa. The biggest problem, however, comes from mixing mathematical and geographical conventions, as e.g. in [5] and [8] and subsequently in further GIS packages.

Insufficient understanding of LSC definition is reflected also in the dimensions attributed to particular LSCs. The dimension [m^{-1}] is natural for LSCs defined by radius of an osculating circle, and for second derivatives of altitude. The latter express change of tangent of inclination over unit distance in a given direction. Expression of the plan curvature of a curve in [$rad.m^{-1}$] (the central angle of a 1 m long circular arch of osculating circle, which is equal to the radius of the circle) is also acceptable. However, in many application studies dimensions are undeclared or poorly declared.

III. Terminology

Absence of a comprehensive classification system of LSCs has led to vagueness in LSC terminology. The same term frequently has various meanings: e.g. for plan curvature, four GIS packages (ArcGIS, SAGA-GIS, GRASS, MICRODEM) give four different results. On the other hand, the same curvature has been given different names in different papers: e.g. the tangential curvature of [6] and [21] is identical with the horizontal curvature of [10]; the profile curvature of [5] is the same as the longitudinal curvature of [14], and so on. Moreover the majority of terms do not fully express the mathematical basis of the specified curvature. Plan curvature is curvature of a contour line, i.e. in plan projection. However, rotor, flowline or streamline curvature [10, 23, 7] also occur in plan projection ('plan slope line curvature'). But flow or stream lines are generally considered as spatial curves that have three various measures of curvature, and rotor curvature is most frequently understand as curvature of a material rotor. Even less specific is the traditional term profile curvature, as we can draw an infinite number of profiles through every point on the land surface. The worst situation is in many application papers where the bare term 'curvature' is used, or terms such as 'general', 'conventional' or 'standard' curvature which have no specified definition. This generally unsystematic labelling of curvatures provides a terminological chaos.

Systematic terminology and labelling of LSCs should contain mathematical meaning yet express functional hierarchy. On higher (broader) hierarchical level we can preserve traditional terms such as plan and profile curvature, for groups of similar curvature; and to specify a subform of the group we can label curvature by combining terms of differential geometry with basic topographic lines, e.g.: normal contour curvature and geodesic contour curvature as subforms of plan curvature. If no geometric curvature is adequate, direction of directional derivative and name of derived field (eventually also order of derivative) can be used: e.g. (2^{nd}) contour derivative of altitude is another subform of the plan curvature group. This permits systematic symbolisation of subforms, e.g. (if c is contour): $(k_n)_c$ and $(k_g)_c$ for normal and geodesic curvatures of contour; and (if z is altitude) z_{cc} for the next subform of plan curvature – second derivative of altitude in direction of contour.

IV. Interpretation

Interpretation of LSCs is the most important aspect for their practical use. About 30 formally defined LSCs are fruitless if we lack efficient interpretation of each one. In the great majority of application papers, the simplest dynamic interpretation of plan and profile curvature is used: plan curvature determines divergence and convergence, profile determines acceleration and deceleration of mass flows. However, if we have four similar curvatures (subforms), that all produce convergence and divergence, what is the difference between them? Many authors ignore this problem: exceptions are e.g. [10] and [28]. Yet little attention is paid to interpretation of Shary's compound curvatures [10] although rare empirical studies confirm their importance [26, 27]. One exception is [28], showing how profile and tangential curvature interact during soil erosion (in the form of mean curvature). [29] presents an attempt to show the importance of difference curvature for definition of potential energy applicable to mass flow on the land surface. This concept can also influence appraisal of the importance of various kinds of plan and profile curvature. If both are sources of change of energy of gravity flows, the possibility of infinite values leads to physical nonsense. As the energy of gravity flows is a finite quantity, curvatures reaching infinity for slope extrema (0° or 90°) are not suitable measures for dynamic interpretation.

Reflection of long-term landform evolution is another essential aspect of interpretation. Curvatures are commonly used in mapping individual landforms, land surface segmentation, and classification. However, an a priori concept of the evolutionary interpretation of LSC has hardly ever been used. The germ of such an interpretation can be found in the concept of elementary forms defined by constant values of altitude, slope, aspect and curvatures [30]. If a tendency to such uniformity really exists, it can be reflected by a concentration of values around zero, in the ascending hierarchical order: altitude – slope / aspect – curvatures – changes of curvature [31]. Various lines of theoretical and empirical evidence show that at least linearity (zero plan and profile curvature) is an attractor of landform development in many cases. We should consider the possibility of interpreting zero values of other LSCs as attractors of landscape development.

Shary's system of LSCs provides a general classification of landforms using LSCs [10]. In the space defined by mean, difference and total (Gaussian) curvature projected onto the plane, where unsphericity curvature = 0, Shary defined twelve main types of landforms (Fig. 1)

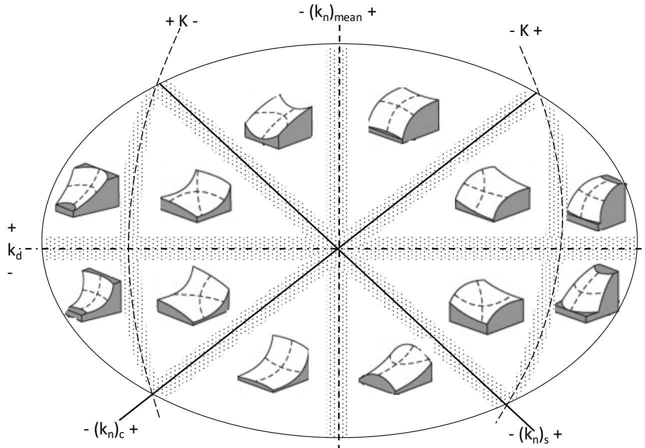

Figure 1. Shary's landform classification. K – total (Gaussian) curvature, $(k_n)_c$ – normal contour (tangential) curvature, $(k_n)_s$ – normal slope line (profile) curvature, k_d – difference curvature. Dotted belts represent the special character of zones along axes – see Fig. 2. Adapted from [32], and modified.

From Shary's statistical hypothesis, there is a 1/12 probability of the random appearance of each landform type in Fig. 1 [10]. As shown in [32], this is generally valid and differences from the statistical hypothesis can be morphogenetically interpreted. However, Shary's assertion that "land form types, for which at least one curvature is zero, are extremely rare; as a rule, they are related to artefacts in elevation matrices" [32, p. 96] is challenged by the elementary forms concept [30]. For this concept, attractors of land surface development are on the axes of the Shary's system (Fig. 2). Cases where a curvature exactly achieves zero value are very exceptional in reality, but we hypothesize a natural tendency to approximate zero values of curvatures: this justifies the importance of classifications containing zero [e.g. 4, 6, 33, 34]. Investigation of any tendency of LSCs to concentrate around zero (as executed e.g. in [31]) results from this theoretical interpretation of LSCs and is an example of using of deeper theoretical knowledge about LSCs in the empiric research.

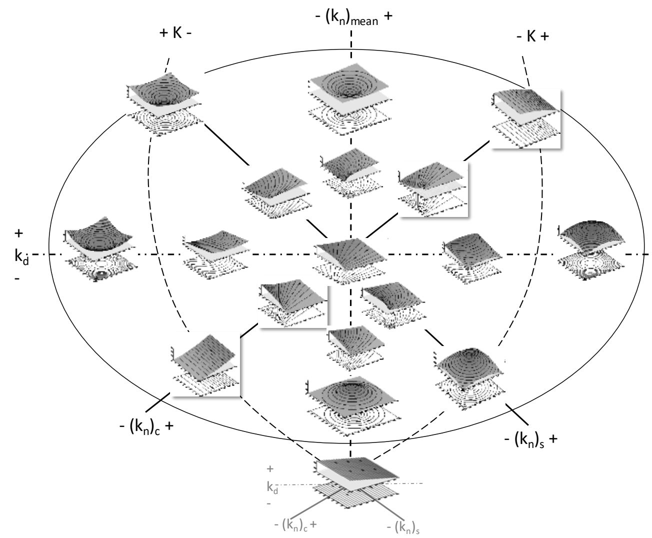

Figure 2. Elementary forms of Minár and Evans [30] positioned in Shary's landform classification. Their position on the axes of Shary's system indicates that zero values of curvatures should be attractors in landform development. Labelling as in Fig. 1.

V. Conclusions

Ongoing confusions in LSC terminology, definitions and interpretations provide an impetus to prepare a comprehensive system of LSCs, unifying terminology and clearly specifying relationships between various types of LSCs. The system should be built on Shary's system of geometric normal curvatures [10], but must go further. It can help to ensure identification of every LSC used in GIS (with definition equations in general form) and in papers (with standardized terminology and labelling).

The place of any curvature in the system should be conditioned not only by clear mathematical definition and relationships with other LSCs, but also by a clear interpretational potential. To develop physically based interpretation of LSCs as both dynamic (influencing distribution of energy on the land surface) and as evolutionary (defining a long term disequilibrium or as attractors of land surface development) is the biggest challenge for the future. The result can be substitution of recent experiments with LSCs which lack a basis in sound theory, by theory-driven investigations as is typical for developed sciences.

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REFERENCES

[1] Evans, I. S. and Minár, J. 2011. A classification of geomorphometric variables. *International Geomorphometry 2011*, 105-108. Geomorphometry.org.

[2] Aandahl, A.R., 1948. The characterization of slope positions and their influence on the total nitrogen content of a few virgin soils of Western Iowa. Soil Science Society of America Proceedings 13, 449–454.

[3] Evans, I.S., 1972. General geomorphometry, derivatives of altitude and descriptive statistics, in: Chorley, R.J. (Ed.), Spatial Analysis in Geomorphology. Methuen, London, pp. 17–90.

[4] Krcho, J., 1973. Morphometric analysis of relief on the basis of geometric aspect of field theory. Acta Geographica Universit. Comenianae, Geographico–physica No 1, pp. 7–233.

[5] Zevenbergen, L.W., Thorne, C.R., 1987. Quantitative analysis of land surface topography. Earth Surface Processes and Landforms 12 (1), 47–56.

[6] Krcho, J., 1990. Morfometrická analýza a digitálne modely georeliéfu., [Morphometrical Analysis and Digital Elevation Models]. Veda, Bratislava (in Slovak).

[7] Shary, P.A., 1991. Topographic method of second derivatives. In: Stepanov, I.N. (Ed.), The Geometry of Earth Surface Structures. Poushchino Scientific Center, Poushchino, pp. 28–58 (in Russian).

[8] Moore, I.D., Grayson, R., Ladson, A., 1991. Digital terrain modelling: A review of hydrological, geomorphological, and biological applications. Hydrol. Process. 5 (1), 3–30.

[9] Jenčo, M., 1992. The morphometric analysis of georelief in terms of a theoretical conception of the complex digital model of georelief. Acta Facultatis Rerum Naturalium Universitatis Comenianae, Geographica 33, 133–153.

[10] Shary, P.A., 1995. Land surface in gravity points classification by a complete system of curvatures. Math. Geol. 27 (3), 373–390.

[11] Hengl, T., Reuter, H.I. (Eds.), 2009. Geomorphometry: Concepts, Software, Applications. Developments in Soil Science 33. Elsevier, Amsterdam.

[12] Florinsky I.V., 2016. Digital Terrain Analysis in Soil Science and Geology, second ed. Elsevier, Amsterdam.

[13] Wilson, J.P., 2018. Environmental Applications of Digital Terrain Modelling. Wiley.

[14] Wood, J.D., 1996. The geomorphological characterisation of digital elevation models. PhD Thesis. University of Leicester, UK.

[15] Pirotti, F., Tarolli, P., 2010. Suitability of LiDAR point density and derived landform curvature maps for channel network extraction. Hydrol. Process. 24, 1187–1197.

[16] Gallant, J.C., Wilson, J.P., 2000. Primary Topographic Attributes, in: Wilson, J.P., Gallant, J. (Eds.), Terrain Analysis: Principles and Applications, Wiley, New York, pp. 51–85.

[17] Koenderink, J.J., van Doorn, A.J., 1992. Surface shape and curvature scales. Image and Vision Computing 10 (8), 557–564.

[18] Krebs, P., Stocker, M., Pezzatti, G.B., Conedera, M. 2015. An alternative approach to transverse and profile terrain curvature. Int. J. Geographical Information Science 29 (4), 643–656.

[19] Cheng, G., Liu, L., Jing, N., Chen, L., Xiong, W., 2012. General-purpose optimization methods for parallelization of digital terrain analysis based on cellular automata. Computers & Geosciences 45, 57–67

[20] Evans, J.S., Hudak, A.T., 2007. A multiscale curvature algorithm for classifying discrete return LiDAR in forested environments. IEEE Transactions on Geoscience and Remote Sensing 45 (4), 1029–1038.

[21] Mitasova, H., Hofierka, J., 1993. Interpolation by regularized spline with tension: II Application to terrain modeling and surface geometry analysis. Mathematical Geology, 25, 657–669.

[22] Hyde, S., Andersson, S., Larsson, K., Blum, Z., Landh, T., Lidin, S., Ninham, B.W., 1997. The Language of Shape. The Role of Curvature in Condensed Matter: Physics, Chemistry and Biology. Elsevier Science

[23] Peckham, S.D., 2011. Profile, Plan and Streamline Curvature: A Simple Derivation and Applications, in: Geomorphometry 2011, Redlands, USA, 7–11 September 2011, pp. 27–30.

[24] Evans, I.S., 1980. An integrated system of terrain analysis and slope mapping. Z. Geomorphol. 36, 274–295.

[25] Krcho, J., Benová, A., 2013. Problém správnej a exaktnej definície geometrických foriem georeliéfu vzhľadom na tiažové pole Zeme., [The problem of correct and exact definition of geometrical forms of georelief regarding the Earth's gravitational field]. Geografický časopis 65 (3), 189–216 (in Slovak).

[26] Mitusov, A., Mitusova, O.E., Wendt, J., Dreibrodt, S., Bork, H.–R., 2014. Correlation of colluvial deposits with the modern land surface and the problem of slope profile description. Geomorphology 220, 30–40.

[27] Mitusov, A.V., Burian, L., Khrisanov, V.R., 2017. Distribution of local landforms at head and end points of gullies on different grid spacing. Catena 159, 159–170.

[28] Mitas, L. and H. Mitasova, 1998. Distributed soil erosion simulation for effective erosion prevention. Water Resources Research, 34 (3): 505-516.

[29] Minár, J., Bandura, P., Holec, J., Popov, A., Gallay, M., Hofierka, J., Kaňuk, J., Drăguţ, L., Evans, I.S., 2018. Physically-based land surface segmentation: Theoretical background and outline of interpretations. PeerJ Preprints 6:e27075v1.

[30] Minár, J. and I.S. Evans, 2008. Elementary forms for land surface segmentation: The theoretical basis of terrain analysis and geomorphological mapping. Geomorphology, 95: 236-259.

[31] Minár, J., Jenčo, M., Evans, I.S., Minár Jr., J., Kadlec, M., Krcho, J., Pacina, J., Burian, L., Benová, A., 2013. Third–order geomorphometric variables (derivatives): definition, computation and utilization of changes of curvatures. Int. J. Geographical Information Science 27 (7), 1381–1402.

[32] Shary, P.A., Sharaya, L.S., Mitusov, A.V., 2005. The problem of scale–specific and scale–free approaches in geomorphometry. Geografia Fisica e Dinamica Quaternaria 28 (1), 81–101.

[33] Richter, H., 1962. Eine neue Methode der grossmassstäbigen Kartierung des Reliefs. Petermanns Geogr. Mitteilungen (Gotha) 106, 309–312.

[34] Dikau, R., 1989. The application of a digital relief model to landform analysis in geomorphology. In: Raper, J. (Ed.), Three-dimensional Applications in Geographical Information Systems. Taylor & Francis, London, pp. 51–77.